Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay

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CISNP13, University of South Carolina
Columbia, U.S.A.
May 19, 2013
Symposium in honor of Frank Avignone and Ettore Fiorini
Google: Frank Avignone; 2008: 588 000;
Google: Frank Avignone; 2013: 2 850 000
Google: Frank Avignone; 2008: 588 000

- Frank T. Avignone, Department of Physics and Astronomy, University of South Carolina 712 Main Street Columbia, South Carolina 29208, (803) 777-6933 ...
  www.physics.sc.edu/FacultyStaff/Bravignone.html

- USC physicist Frank Avignone
  Physics professor Frank Avignone joined USC faculty 40 years ago, but it will be another decade, he hopes, before you read about his retirement ...
  www.sc.edu/usctimes/articles/2005-02/frank.avignone.html

- [0711.4808] The MAJORANA 76Ge neutrino less double-beta decay ...
  From: Frank Avignone III Thu, 29 Nov 2007 19:40:56 GMT (776kb). Link back to: arXiv, ...
  export.arxiv.org/abs/0711.4808

- LBNL CUORE Group
  Frank T. Avignone (USC), (803)777-6933, Avignone@sc.edu. Ettore Fiorini* (Milan University), 39 02 64482432,2424 / 2463 (fax), ettore.fiorini@mib.infn.it ...
  www-rnc.lbl.gov/ nxu/cuore/cuore.lbl.html

- Axions
  Frank T. Avignone (USC) and W. Haxton (UW). ... K. Zioutas, C.E. Aalseth, D. Abriola, F.T. Avignone, R.L. Brodzinski, J.I. Collar, R. Creswick, ...
  collargroup.uchicago.edu/projects/axion/index.html

- Supernova science at spallation neutron sources discussions with Frank Avignone, John Beacom, Jeff Blackmon, Dick Boyd, David Dean, Yuri Efremenko, Jon Engel, George Fuller, Wick Haxton, Ken Lande, ...
  www.iop.org/EJ/article/0954-3899/29/11/008/g3.11.008.pdf
TAUP 2003 - Convener Contacts
Frank Avignone, waters@psc.psc.sc.edu; Hiro Ejiri, ejiri@rcnpax.rcnp.osaka-u.ac.jp Jouni Suhonen, suhonen@phys.jyu.fi int.phys.washington.edu/taup2003/contacts.html

SUMMARY of the 1 MEETING OF THE LSC SCIENTIFIC COMMITTEE Hotel ... Aprile, Frank Avignone (Chair), Laura Baudis, Yves Declais, Juan Fuster, .... Juan Fuster and Frank Avignone. ezpc00.unizar.es/lsc/LSC-MINSC1-08.pdf

ORAU News: Frank Avignone Receives ORAU Outstanding Leadership Award
Frank T. Avignone III, Carolina Endowed Professor of Physics and Astronomy at the University of South Carolina, received the Oak Ridge Associated ... www.orau.org/news/releases/2003/fy03-14.htm

Fulbright US Scholar Directory: Alphabetical Index Avignone, Frank Titus III; Physics and Astronomy; Italy. B. Baar, Kenneth K.; Architecture; Albania. Baer, Adela S.; Biological Sciences; Malaysia ... www.cies.org/schlr-directories/usdir01/us-dir-name.htm

CD Baby: MELISSA FAHN: F. Avignon The nine songs on "F. Avignon" make up a song cycle that takes the listener on a dynamic ... guitarists Chris Clermont and Jamie Findlay, bassists Derek Frank, ... cdbaby.com/cd/melissafahn

The Transporter Script - transcript from the screenplay and/or...
"We need you Avignone to take us to ... The deal was this far and no further. .... You're breaking the rules, Frank. Not good to break the rules. ...” www.script-o-rama.com/movie-scripts/t/transporter-script-transcript-jason-statham.html

Francia - Lotte e repressione (2007). Un punto sulla situazione ... Due ... Avignone, accusati dell'incendio della sede del .... a Parigi Frank ed Ines, sono fermati il 23 ...
Google: Frank Avignone; 2013: 2 850 000


- Invited Speakers - PNNL: Dark Matter Silver Jubilee (June 19 - 21, 2012) - The Symposium events.pnnl.gov/jubilee/speakers.stm. Speaker: Frank T. Avignone III. Time: 4:00pm. Place: Battelle ...

- NnuMass 2013 - Participants - Infn artico.mib.infn.it/numass2013/index.php/participants. Frank Avignone, Department of Physics and Astronomy University of South ...

- Scientific Program - TAUP2013 - Berkeley Lab Commons commons.lbl.gov TAUP2013. Plenary Session I: Cosmology 8:45-10:20. - Welcome to TAUP 2013 (5 minutes). - Wick Haxton, UC Berkeley/LBNL, and Frank Avignone, Univ. South Carolina ...

- Committees-Neutrino 2012. Frank T. Avignone III (South Carolina) Alesandro Bettini (Canfranc Underground)
On June 21, 2010, the University of Zaragoza in Spain conferred an honorary doctorate on Professor Frank Avignone in recognition of his many contributions to the field of particle astrophysics. The formal ceremony followed a 500-year-old format, in which he gave his acceptance speech in Spanish. This was Avignone's second honorary doctoral degree, the first one having been bestowed by the University of Buenos Aires in 2004. Avignone joined the USC faculty in January 1965, and although he retired as a state employee in 1998, he has worked fulltime under contract at USC as a research professor ever since. He continues to hold the title of Carolina Endowed Professor of Physics and Astronomy and is the leader of the Particle Astrophysics Group. He served as chair of the department from May 1979 until he took his new position in July 1998.

Philanthropy: The 100 Day Difference — Trauma Services of the Midlands
www.stsm.org/100daydifference Frank Avignone III. Penelope Arnold (in honor of Emily Lloyd Flores). Heyward Bannister. Sue Berkowitz. Michelle Kevin Brown. Penny Moss Blachman ...

Letter to the UNC Faculty
- Help Paul Frampton helppaulframpton.org/lettertofaculty.pdf -Frank Avignone, Carolina Endowed Professor of Physics and Astronomy, USC. -Julia T. Wood, Lineberger Professor of Humanities Emerita ...

Property valuation of Trenholm Road, Arcadia Lakes, SC: 6607 ...
www.city-data.com/richland-county/T/Trenholm-Road-1.html - Cached Owner: ANN MARIE BRAITHWAITE and TITUS AVIGNONE /FRANK III Total land value: 84,000. Total value for property: 449,400. Tax assessed: 17,980 ...
Avignon Pharmacy

Home/Fashion/Beauty/Avignon Pharmacy.
Google: Ettore Fiorini; 2008: 63 800
Google: Ettore Fiorini; 2013: 948 000
Universita degli Studi di Milano - Bicocca - FIORINI ETTORE FIORINI ETTORE. professore ordinario. Settore scientifico disciplinare: Settore FIS/04 - Fisica Nucleare E Subnucleare. Dipartimento: ... www.unimib.it/go/Home/Pagine-Speciali/Elenco-Docenti/FIORINI-ETTORE

Current Academic Responsibilities:

Direttore dei Laboratori di Radioattivita' e di Criogenia.

Other Professional Responsibilities:

Responsabile nazionale degli esperimenti "Mibeta" e "Cuore". Coordinatore del Network Europeo sui Rivelatori Termici.

Particle physicists plumb the depths for Roman lead - 13 July 1991 ...
The physicists, Gianni Fiorentini and Ettore Fiorini, want the lead for experiments that are of critical importance in particle physics and cosmology. ...

Premio Enrico Fermi 2007 della Societa Italiana di Fisica
Motivazioni: a Ettore Fiorini per il contributo alla scoperta delle correnti deboli neutre e allo studio dei neutrini solari. www.sif.it/SIF/it/portal/attivita/concorsi

Ortvay Kollokvium
Ettore Fiorini (Dipt. di Fisica, Universita Milano-Bicocca) (Receipient of the Marx Medal): "The neutrinoless beta decay and the nature of neutrino" ...

• IUPAP COMMISSION C12: 1999-2002
E-mail: Ettore.Fiorini@MI.infn.it. JONSON, B. (1996) Department of Physics, Chalmers University of Technology, SE-412 96 Goeteborg, SWEDEN. ...

• Peer review committee
Ettore Fiorini. INFN Milan. ettore.fiorini@mib.infn.it.
G. Smadja. IPN Lyon. g.smadja@ipnl.in2p3.fr.
Joe Silk. Nuclear and Astrophysics Laboratory ...
appec.in2p3.fr/pages/peer.htm
COMUNCATO STAMPA (Press release) 11 febbraio 2008
“L’occhio” nucleare rivela: Napoleone non è stato avvelenato”
“The nuclear “eye” reveals: Napoleon was not poisoned”
11 feb 2008 Ettore Fiorini, docente di Fisica Nucleare all’ Università Milano Bicocca e e-mail: ettore.fiorini@mib.infn.it. Ezio Previtali
www.ricercaitaliana.it/stdoc/pdfnapoleon.pdf

FOXNews.com - Arsenic Poisoning Ruled Out in Napoleon's Death ...
Feb 13, 2008 The researchers, including Ettore Fiorini of the Italian Na-
tional Institute of Nuclear Physics and the University of Milano-Biccoca, ...
www.foxnews.com/story/0,2933,330450,00.html

Napoleon didn’t die from arsenic poisoning - Telegraph
Drs Ettore Fiorini and Ezio Previtali of INFN, who did the study with Angela
Santagostino of the University of Milan at a small nuclear reactor at the ...
www.telegraph.co.uk/earth/main.jhtml?xml=/earth/2008/02/11/scinap111.xml

Il Sole 24 ORE: finanza, economia, esteri, valute, borsa e fisco
Ettore Fiorini, ordinario di Fisica nucleare e subnucleare all’Universita‘ Milano Bicocca,
che da molti anni studia i processi rari legati alla radiazione ... www.ilsole24ore.com

Photo Blog Pull Out Ravenna
Ettore ... il mitico ETTORE !!! ... FIORINI si stira !!!
JOIN THE GANG 15.03.2008 !!! www.pulloutfoto.splinder.com
• Ancient Romans join neutrino hunt - physicsworld.com physicsworld.com/cws/article/news/42445 Apr 23, 2010 When nuclear physicist Ettore Fiorini at the University of Milan-Bicocca read about the find in a newspaper he went to Cagliari to offer the ... Ever on the look-out for ultra-low radioactive materials to shield their sensitive experiments, nuclear physicists have struck gold with a consignment of lead that lay on the floor of the Mediterranean Sea for 2000 years. The almost completely inert ancient lead will be used to line the CUORE neutrino experiment located under the Gran Sasso mountain in central Italy.

• Neutrinoless Double Beta Decay - Neutrino Unbound - Infn www.nu.to.infn.it/Neutrinoless Double Beta Decay/ [1-3]: Neutrino physics with cryogenic detectors, Fiorini, Ettore, Prog. Part. Nucl. Phys. 64 (2010) 241-248. [1-4]: Liquid Xenon Detectors for Particle Physics and ...


• Double beta decay: yesterday, today, tomorrow AIP Conference ... link.aip.org/link/APCPCS/1417/47/1 by E Fiorini - 2011 - Cited by 2 - Related articles Ettore Fiorini. Abstract. After a brief introduction on the main features of Double Beta Decay (DBD) and on its origin, its importance is stressed in view of the ...

• Measurement of airborne 131I, 134Cs, and 137Cs nuclides due to ... arxiv.org by M Clemenza - 2011 - Cited by 3 - Related articles Jun 21, 2011 Massimiliano Clemenza,
Ettore Fiorini, Ezio Previtali, Elena Sala. After the earthquake and the tsunami occurred in Japan on 11th March 2011, ...

• Neutrino physics: Beta test : Nature News Comment www.nature.com/news/neutrino-physics-beta-test-1.10988 Jul 12, 2012 A definitive sighting, says Ettore Fiorini, a particle physicist at the University of Milano-Bicocca in Italy, would be one of the most important ...

• Now 2010- Program - INFN Bari www.ba.infn.it/ now/now2010/Program/program.html Chair: Ettore Fiorini. Time, Speaker, Title. 8.30 - 8.40, Gianluigi Fogli, Welcome to NOW 2010 (pptx). 8.40 - 9.15, Francesco Iachello, Advances in the theory of ...


• Committees - Neutrino 2012 neu2012.kek.jp/neu2012/committees.html Ettore Fiorini (Milano) Francis Halzen (Wisconsin) Kunio Inoue (Tohoku) Takaaki Kajita (ICRR Tokyo) Stavros Katsanivas (Lyon) Soo-Bong Kim (Seoul) ...

• Ettore Fiorini vince il Premio Bruno Pontecorvo - Università degli ... www.unimib.it Feb 21, 2013 Il professor Fiorini riceverà il premio istituito dal Joint Institute for Nuclear Research per le ricerche condotte nell’ambito della fisica delle ...
Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay

Based on:
If the decay \((A, Z) \rightarrow (A, Z + 2) + e^- + e^- ((\beta\beta)_{0\nu}\text{-decay})\) will be observed, the question will inevitably arise:

Which mechanism is triggering the decay?

How many mechanisms are involved?

“Standard Mechanism”: light Majorana \(\nu\) exchange.

Fundamental parameter - the effective Majorana mass:

\[
\langle m \rangle = \sum_j^{\text{light}} (U_{ej})^2 m_j , \text{ all } m_j \geq 0 ,
\]

\(U\) - the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix, \(m_j\) - the light Majorana neutrino masses, \(m_j \lesssim 1\) eV.

\(U\) - CP violating, in general: \((U_{ej})^2 = |U_{ej}|^2 e^{i\alpha_j}\), \(j = 2, 3, \alpha_{21}, \alpha_{31}\) - Majorana CPV phases.

S.M. Bilenky, J. Hosek, S.T.P., 1980
Nuclear $0\nu\beta\beta$-decay

strong in-medium modification of the basic process

$$dd \rightarrow uue^{-}e^{-}(\bar{\nu}_{e}\bar{\nu}_{e})$$

virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus
\[ \sin^2 \theta_{13} = 0.0236 \pm 0.0042, \quad \delta = 0; \quad \alpha_{21}, \alpha_{31} \text{ - varied in the interval } [0, \pi]; \]
\[ \Delta m_{21}^2 = 7.58 \times 10^{-5} \text{ eV}^2, \quad 1\sigma(\Delta m_{21}^2) = 3.5\%; \]
\[ \sin^2 \theta_{21} = 0.306, \quad 1\sigma(\sin^2 \theta_{21}) = 6\%; \]
\[ |\Delta m_{31}^2| = 2.35 \times 10^{-3} \text{ eV}^2, \quad 1\sigma(|\Delta m_{31}^2|) = 5\%. \]


\[ 2\sigma(|<m>|) \text{ } \text{used.} \]
A number of different mechanisms possible. For a given mechanism \( \kappa \) we have in the case of 
\((A, Z) \rightarrow (A, Z + 2) + e^- + e^-:\)

\[
\frac{1}{T_{1/2}^{0\nu}} = |\eta_{\kappa}^{LNV}|^2 \; G^{0\nu}(E_0, Z)|M'_{\kappa}^{0\nu}|^2,
\]

\(\eta_{\kappa}^{LNV}\) - the fundamental LNV parameter characterising the mechanism \( \kappa \),

\(G^{0\nu}(E_0, Z)\) - phase-space factor (includes \(g_A^4 = (1.25)^4\), as well as \(R^{-2}(A)\), \(R(A) = r_0 A^{1/3}\) with \(r_0 = 1.1 \; fm\)),

\(M'_{\kappa}^{0\nu} = (g_A/1.25)^2 M_{\kappa}^{0\nu}\) - NME (includes \(R(A)\) as a factor).
Different Mechanisms of \( (\beta\beta)_{0\nu}\)-Decay

\begin{align*}
\text{Light Majorana Neutrino Exchange} \\
\eta_\nu &= \frac{\langle m \rangle}{m_e}.
\end{align*}

\text{Heavy Majorana Neutrino Exchange Mechanisms} \\
(V-A) \text{ Weak Interaction, LH } N_k, M_k \gtrsim 10 \text{ GeV:} \\
\eta_{N}^L &= \sum_{k}^{\text{heavy}} U_{ek}^2 \frac{m_p}{M_k}, m_p \text{ - proton mass, } U_{ek} \text{ - CPV.}
(V+A) Weak Interaction, RH $N_k, M_k \gtrsim 10$ GeV:

$$\eta^R_N = \left( \frac{M_W}{M_{WR}} \right)^4 \sum_{k \text{ heavy}} V^2_{ek} \frac{m_p}{M_k}; \ V_{ek}: N_k - e^- \text{ in the CC .}$$

$M_W \approx 80$ GeV; $M_{WR} \gtrsim 2.5$ TeV; $V_{ek}$ - CPV, in general.

**A comment.**

(V-A) CC Weak Interaction:

$$\bar{e}(1 + \gamma_5)e^c \equiv 2\bar{e}_L (e^c)_R , \ e^c = C(\bar{e})^T ,$$

$C$ - the charge conjugation matrix.

(V+A) CC Weak Interaction:

$$\bar{e}(1 - \gamma_5)e^c \equiv 2\bar{e}_R (e^c)_L .$$

**The interference term: $\propto m_e$, suppressed.**

SUSY Models with R-Parity Non-conservation

\[ \mathcal{L}_{R_p} = \chi'_{111} \left[ (\bar{u}_L \bar{d}_L) \left( \begin{array}{c} e^c_R \\ -\nu^c_{eR} \end{array} \right) \bar{d}_R + (\bar{e}_L \bar{\nu}_{eL}) d_R \left( \begin{array}{c} \bar{u}^*_L \\ -\bar{d}^*_L \end{array} \right) + (\bar{u}_L \bar{d}_L) d_R \left( \begin{array}{c} \bar{e}^*_L \\ -\bar{\nu}^*_{eL} \end{array} \right) \right] \]
The Gluino Exchange Dominance Mechanism

\[ \eta\lambda' = \frac{\pi\alpha_s}{6} \frac{\lambda_1' \lambda_1' \lambda_1' \lambda_1'}{G_F m^4 d_R m\bar{g}} \left[ 1 + \left( \frac{m_d R}{m\bar{u}_L} \right)^2 \right]^2, \]

\[ G_F \text{ - the Fermi constant, } \alpha_s = \frac{g_3^2}{4\pi}, g_3 \text{ - the SU}(3)_c \text{ gauge coupling constant, } m\bar{u}_L, m\bar{d}_R \text{ and } m\bar{g} \text{ - the masses of the LH u-squark, RH d-squark and gluino.} \]

The Squark-Neutrino Mechanism

\[ \eta\tilde{q} = \sum_k \frac{\lambda_1' \lambda_1' \lambda_1' \lambda_1'}{2\sqrt{2} G_F} \sin 2\theta_d(k) \left( \frac{1}{m_{\tilde{d}_1(k)}^2} - \frac{1}{m_{\tilde{d}_2(k)}^2} \right), \]

\( d(k) = d, s, b; \theta_d: \tilde{d}_{kL} - \tilde{d}_{kR} \) - mixing (3 light Majorana neutrinos assumed).

The 2\(e^-\) current in both mechanisms:

\[ \bar{e}(1 + \gamma_5)e^c \equiv 2\bar{e}_L(e^c)_R, \text{ as in the “standard” mechanism.} \]
Example: $\beta_0^\nu$- Decay and TeV Scale See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos $N_j$ at the TeV scale:

$$m_\nu \simeq - M_D \hat{M}_N^{-1} M_D^T,$$

$\hat{M} = \text{diag}(M_1, M_2, M_3)$, $M_j \sim (100 - 1000)$ GeV.

$$\mathcal{L}_{NC}^N = - \frac{g}{2\sqrt{2}} \bar{\nu}_\ell \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k W^\alpha + \text{h.c.},$$

The exchange of virtual $N_j$ gives a contribution to $|<m>|$:

$$|<m>| \cong \left| \sum_i (U_{PMNS})_{ei}^2 m_i - \sum_k f(A, M_k) (RV)_{ek}^2 (0.9 \text{ GeV})^2 \right|,$$

$$f(A, M_k) \cong f(A).$$

For, e.g., $^{48}$Ca, $^{76}$Ge, $^{82}$Se, $^{130}$Te and $^{136}$Xe, the function $f(A)$ takes the values $f(A) \cong 0.033$, 0.079, 0.073, 0.085 and 0.068, respectively.

- All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos $N_{1,2}$, which form a pseudo-Dirac pair:
  $$M_2 = M_1(1 + z), \ 0 < z \ll 1.$$

- Only NH and IH $\nu$ mass spectra possible.

- The Predictions for $|<m>|$ can be modified considerably.

$|<m>|$ vs $|(RV)_{e1}|$ for $^{76}$Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100 \text{ GeV}$ and $i)$ $y = 0.001$ (blue), $ii)$ $y = 0.01$ (green). The gray markers correspond to $|<m>^{\text{std}}| = \sum_i (U_{PMNS})_{ei}^2 m_i$.

Illustrative examples:

\[ T^{0\nu}_{1/2}(^{76}\text{Ge}), \ T^{0\nu}_{1/2}(^{100}\text{Mo}), \ T^{0\nu}_{1/2}(^{130}\text{Te}) \] used as input,

\[ T^{0\nu}_{1/2}(^{76}\text{Ge}) \geq 1.9 \times 10^{25} \text{y}, \ T^{0\nu}_{1/2}(^{76}\text{Ge}) = 2.23^{+0.44}_{-0.31} \times 10^{25} \text{y} \]

(lower limit: Heidelberg-Moscow collab., 2001; value - Klapdor-Kleingrothaus et al., 2004.)

\[ 5.8 \times 10^{23} \text{y} \leq T^{0\nu}_{1/2}(^{100}\text{Mo}) \leq 5.8 \times 10^{24} \text{y} \]  (lower limit - NEMO3)

\[ 3.0 \times 10^{24} \text{y} \leq T^{0\nu}_{1/2}(^{130}\text{Te}) \leq 3.0 \times 10^{25} \text{y} \]  (lower limit-CUORICINO)

Constraints from \(^3\text{H} \ \beta\)-decay data

Light \(\nu\) exchange + “nonstandard” mechanisms

Moscow, Mainz: \(m(\bar{\nu}_e) < 2.3 \text{ eV}; \ |\eta_\nu|^2 \times 10^{10} < 0.21 \).

KATRIN: \(m(\bar{\nu}_e) < 0.2 \text{ eV}; \ |\eta_\nu|^2 \times 10^{10} < 1.6 \times 10^{-3} \).
Calculation of the NMEs for $^{76}Ge$, $^{82}Se$, $^{100}Mo$, $^{130}Te$

The NME: obtained within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA) (takes into account the Pauli exclusion principle and conserves the mean particle number in correlated ground state).

Two choices of single-particle basis used:

i) the intermediate size model space has 12 levels (oscillator shells N=2-4) for $^{76}Ge$ and $^{82}Se$, 16 levels (oscillator shells N=2-4 plus the f+h orbits from N=5) for $^{100}Mo$ and 18 levels (oscillator shells N=3,4 plus f+h+p orbits from N=5) for $^{130}Te$;

ii) the large size single particle space contains 21 levels (oscillator shells N=0-5) for $^{76}Ge$, $^{82}Se$ and $^{100}Mo$, and 23 levels for $^{130}Te$ (N=1-5 and i orbits from N=6).

The single particle energies: obtained by using a Coulomb–corrected Woods–Saxon potential. Two-body G-matrix elements we derived from the Argonne and the Charge Dependent Bonn (CD-Bonn) one-boson exchange potential within the Brueckner theory. The calculations: for $g_{ph} = 1.0$. The particle-particle strength parameter $g_{pp}$ of the SRQRPA is fixed by the data on the two-neutrino double beta decays.

Table

The phase-space factor $G^{0\nu}(E_0, Z)$ and the nuclear matrix elements $M^{0\nu}_{\nu}$ (light Majorana neutrino exchange mechanism), $M^{0\nu}_{N}$ (heavy Majorana neutrino exchange mechanism), $M^{0\nu}_{\chi}$ (mechanism of gluino exchange dominance in SUSY with trilinear R-parity breaking term) and $M^{0\nu}_{q}$ (squark-neutrino mechanism) for the $(\beta\beta)_{0\nu}$-decays of $^{76}Ge$, $^{100}Se$, $^{100}Mo$ and $^{130}Te$. The nuclear matrix elements were obtained within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA).
| Nuclear transition | $G^{0\nu}(E_0, Z)$ $[y^{-1}]$ | $|M^{0\nu}\rangle$ | $|M^{0\nu}_N\rangle$ | $|M^{0\nu}_\lambda\rangle$ | $|M^{0\nu}_{\tilde{q}}\rangle$ |
|--------------------|-------------------------------|-----------------|-----------------|-----------------|-----------------|
| $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ | $7.98 \times 10^{-15}$ | Argonne intm. | 3.85 172.2 387.3 396.1 | 4.75 232.8 587.2 594.3 | 1.0 1.0 1.0 1.0 |
| | | large | 4.39 196.4 461.1 476.2 | 5.44 264.9 699.6 717.8 | 1.25 1.25 1.25 1.25 |
| | | CD-Bonn intm. | 4.15 269.4 339.7 408.1 | 5.11 351.1 514.6 611.7 | 1.0 1.0 1.0 1.0 |
| | | large | 4.69 317.3 392.8 482.7 | 5.82 411.5 595.6 727.6 | 1.25 1.25 1.25 1.25 |
| $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ | $3.53 \times 10^{-14}$ | Argonne intm. | 3.59 164.8 374.5 379.3 | 4.54 225.7 574.2 577.9 | 1.0 1.0 1.0 1.0 |
| | | large | 4.18 193.1 454.9 465.1 | 5.29 262.9 697.7 710.2 | 1.25 1.25 1.25 1.25 |
| | | CD-Bonn intm. | 3.86 258.7 328.7 390.4 | 4.88 340.4 503.7 594.5 | 1.0 1.0 1.0 1.0 |
| | | large | 4.48 312.4 388.0 471.8 | 5.66 408.4 594.4 719.9 | 1.25 1.25 1.25 1.25 |
| $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$ | $5.73 \times 10^{-14}$ | Argonne intm. | 3.62 184.9 412.0 405.1 | 4.39 249.8 629.4 612.1 | 1.0 1.0 1.0 1.0 |
| | | large | 3.91 191.8 450.4 449.0 | 4.79 259.8 690.3 682.6 | 1.25 1.25 1.25 1.25 |
| | | CD-Bonn intm. | 3.96 298.6 356.3 415.9 | 4.81 388.4 543.7 627.9 | 1.0 1.0 1.0 1.0 |
| | | large | 4.20 310.5 384.4 454.8 | 5.15 404.3 588.6 690.5 | 1.25 1.25 1.25 1.25 |
| $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ | $5.54 \times 10^{-14}$ | Argonne intm. | 3.29 171.6 385.1 382.2 | 4.16 234.1 595.2 588.9 | 1.0 1.0 1.0 1.0 |
| | | large | 3.34 176.5 405.5 403.1 | 4.18 239.7 626.0 620.4 | 1.25 1.25 1.25 1.25 |
| | | CD-Bonn intm. | 3.64 276.8 335.8 396.8 | 4.62 364.3 518.8 611.1 | 1.0 1.0 1.0 1.0 |
| | | large | 3.74 293.8 350.1 416.3 | 4.70 384.5 540.3 640.7 | 1.25 1.25 1.25 1.25 |
Important feature of the NMEs

For each mechanism \( \kappa \) discussed, the NMEs for the nuclei considered differ relatively little:

\[
|M'_{\kappa i} - M'_{\kappa j}| << M'_{\kappa i}, M'_{\kappa j}, \text{ typically}
\]

\[
\frac{|M'_{\kappa i} - M'_{\kappa j}|}{0.5(M'_{\kappa i} + M'_{\kappa j})} \sim 0.1, \ i \neq j = ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te.
\]
Two “Non-Interfering” Mechanisms

Example: light LH and heavy RH Majorana ν exchanges

The corresponding LNV parameters, |\(\eta_\nu\)| and |\(\eta_R\)| - from “data” on \(T_{1/2}^{0\nu}\) of two nuclei:

\[
\frac{1}{T_1 G_1} = |\eta_\nu|^2 |M_{1,\nu}'0\nu|^2 + |\eta_R|^2 |M_{1,N}'0\nu|^2,
\]

\[
\frac{1}{T_2 G_2} = |\eta_\nu|^2 |M_{2,\nu}'0\nu|^2 + |\eta_R|^2 |M_{2,N}'0\nu|^2.
\]

The solutions read:

\[
|\eta_\nu|^2 = \frac{|M_{2,N}'0\nu|^2/T_1 G_1 - |M_{1,N}'0\nu|^2/T_2 G_2}{|M_{1,\nu}'0\nu|^2|M_{2,N}'0\nu|^2 - |M_{1,N}'0\nu|^2|M_{2,\nu}'0\nu|^2},
\]

\[
|\eta_R|^2 = \frac{|M_{1,\nu}'0\nu|^2/T_2 G_2 - |M_{2,\nu}'0\nu|^2/T_1 G_1}{|M_{1,\nu}'0\nu|^2|M_{2,N}'0\nu|^2 - |M_{1,N}'0\nu|^2|M_{2,\nu}'0\nu|^2}.
\]

Solutions giving |\(\eta_\nu\)|\(^2\) < 0 and/or |\(\eta_R\)|\(^2\) < 0 are unphysical. Given a pair \((A_1, Z_1), (A_2, Z_2)\) of the three \(^{76}\text{Ge}\), \(^{100}\text{Mo}\) and \(^{130}\text{Te}\) we will be considering, and \(T_1\), and choosing (for convenience) always \(A_1 < A_2\), positive solutions for |\(\eta_\nu\)|\(^2\) and |\(\eta_R\)|\(^2\) - possible for the following range of values of \(T_2\):
The positivity conditions

\[
\frac{T_1 G_1 |M'_{1,\nu, N}|^2}{G_2 |M'_{2,\nu, N}|^2} \leq T_2 \leq \frac{T_1 G_1 |M'_{1,\nu}|^2}{G_2 |M'_{2,\nu}|^2}
\]

(|M'_{1,\nu}|^2/|M'_{2,\nu}|^2 > |M'_{1,\nu, N}|^2/|M'_{2,\nu, N}|^2) (from Table 1) used.

Using \(G_1, 2\), and \(M'_{i,\nu, N}, M'_{i,\nu}, i = 1, 2\), (Table 1, “CD-Bonn, large, \(g_A = 1.25 (1.0)\)”), we get the positivity conditions for the 3 ratios of pairs of \(T_{1/2}^{0\nu}\):

\[
0.15 \leq \frac{T_{1/2}^{0\nu}(100 \text{Mo})}{T_{1/2}^{0\nu}(76 \text{Ge})} \leq 0.18 (0.17),
\]

\[
0.17 \leq \frac{T_{1/2}^{0\nu}(130 \text{Te})}{T_{1/2}^{0\nu}(76 \text{Ge})} \leq 0.22 (0.23),
\]

\[
1.14 (1.16) \leq \frac{T_{1/2}^{0\nu}(130 \text{Te})}{T_{1/2}^{0\nu}(100 \text{Mo})} \leq 1.24 (1.30).
\]
Similar results with Argonne, large, $g_A=1.25(1.0)$ NMEs:

\[
0.15 \leq \frac{T_{1/2}^{0\nu}(^{100}Mo)}{T_{1/2}^{0\nu}(^{76}Ge)} \leq 0.18,
\]
\[
0.18 \leq \frac{T_{1/2}^{0\nu}(^{130}Te)}{T_{1/2}^{0\nu}(^{76}Ge)} \leq 0.24 \ (0.25),
\]
\[
1.22 \leq \frac{T_{1/2}^{0\nu}(^{130}Te)}{T_{1/2}^{0\nu}(^{100}Mo)} \leq 1.36 \ (1.42).
\]

The physical solutions possible only for remarkably narrow intervals of $T_2/T_1$. If any of the ratios is shown to lie outside the relevant intervals, the case - excluded.

Conditions for only one mechanism being active:

\[
|\eta_R|^2 = 0 : \ |M'_{1,\nu}^{0\nu}|^2 T_1 G_1 = |M'_{2,\nu}^{0\nu}|^2 T_2 G_2,
\]
\[
|\eta_N|^2 = 0 : \ |M'_{1,N}^{0\nu}|^2 T_1 G_1 = |M'_{2,N}^{0\nu}|^2 T_2 G_2.
\]
Comments.

- The feature discussed above - common to all cases of two “non-interfering” mechanisms considered.
- The indicated specific half-life intervals for the various isotopes, are stable with respect to the change of the NMEs.
- The intervals of \( T_2/T_1 \) depend on the type of the two “non-interfering” mechanisms. However, the differences in the cases of the exchange of heavy Majorana neutrinos coupled to \((V+A)\) currents and i) light Majorana neutrino exchange, or ii) the gluino exchange mechanism, or iii) the squark-neutrino exchange mechanism, are extremely small.
- One of the consequences - if it will be possible to rule out one of them as the cause of \( (\beta\beta)_{0\nu} \)-decay, most likely one will be able to rule out all three of them.
- Using the indicated difference to get information about the specific pair of “non-interfering” mechanisms possibly operative in \( (\beta\beta)_{0\nu} \)-decay requires, in the cases considered by us, an extremely high precision in the measurement of the \( (\beta\beta)_{0\nu} \)-decay half-lives of the isotopes considered. The levels of precision required seem impossible to achieve in the foreseeable future.
- If it is experimentally established that any of the indicated intervals of half-lives lies outside the interval of physical solutions of \(|\eta_\nu|^2\) and \(|\eta_R|^2\), obtained taking into account all relevant uncertainties, one would be led to conclude that the \( (\beta\beta)_{0\nu} \)-decay is not generated by the two mechanisms considered.
- The constraints under discussion will not be valid, in general, if the \( (\beta\beta)_{0\nu} \)-decay is triggered by two “interfering” mechanisms with a non-negligible (destructive) interference term, or by more than two mechanisms none of which plays a subdominant role in \( (\beta\beta)_{0\nu} \)-decay.
The predictions for the half-life of a third nucleus \((A_3, Z_3)\), using as input in the system of equations for \(|\eta_\nu|^2\) and \(|\eta_R|^2\) the half-lives of two other nuclei \((A_1, Z_1)\) and \((A_2, Z_2)\). The three nuclei used are \(^{76}\text{Ge}\), \(^{100}\text{Mo}\) and \(^{130}\text{Te}\). The results shown are obtained for a fixed value of the half-life of \((A_1, Z_1)\) and assuming the half-life of \((A_2, Z_2)\) to lie in a certain specific interval. The physical solutions for \(|\eta_\nu|^2\) and \(|\eta_R|^2\) are then used to derive predictions for the half-life of the third nucleus \((A_3, Z_3)\). The latter are compared with the existing experimental lower limits. The results - obtained with “CD-Bonn, large, \(g_A = 1.25\)” NMEs (Table 1). One star beside the isotope pair whose half-lives are used as input indicates predicted ranges of half-lives of the nucleus \((A_3, Z_3)\) that are not compatible with the existing lower bounds.

<table>
<thead>
<tr>
<th>Pair</th>
<th>(T_{1/2}^{\nu}(A_1, Z_1)) [yr]</th>
<th>(T_{1/2}^{\nu}(A_2, Z_2)) [yr]</th>
<th>Prediction on ([A_3, Z_3]) [yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{76}\text{Ge} –^{100}\text{Mo})</td>
<td>(T(\text{Ge}) = 2.23 \cdot 10^{25})</td>
<td>(3.23 \cdot 10^{24} \leq T(\text{Mo}) \leq 3.97 \cdot 10^{24})</td>
<td>(3.68 \cdot 10^{24} \leq T(\text{Te}) \leq 4.93 \cdot 10^{24})</td>
</tr>
<tr>
<td>(^{76}\text{Ge} –^{130}\text{Te})</td>
<td>(T(\text{Ge}) = 2.23 \cdot 10^{25})</td>
<td>(3.68 \cdot 10^{24} \leq T(\text{Te}) \leq 4.93 \cdot 10^{24})</td>
<td>(3.23 \cdot 10^{24} \leq T(\text{Mo}) \leq 3.97 \cdot 10^{24})</td>
</tr>
<tr>
<td>(^{76}\text{Ge} –^{100}\text{Mo})</td>
<td>(T(\text{Ge}) = 10^{26})</td>
<td>(1.45 \cdot 10^{25} \leq T(\text{Mo}) \leq 1.78 \cdot 10^{25})</td>
<td>(1.65 \cdot 10^{25} \leq T(\text{Te}) \leq 2.21 \cdot 10^{25})</td>
</tr>
<tr>
<td>(^{100}\text{Mo} –^{130}\text{Te})</td>
<td>(T(\text{Mo}) = 5.8 \cdot 10^{23})</td>
<td>(6.61 \cdot 10^{23} \leq T(\text{Te}) \leq 7.20 \cdot 10^{23})</td>
<td>(1.45 \cdot 10^{25} \leq T(\text{Mo}) \leq 1.78 \cdot 10^{25})</td>
</tr>
<tr>
<td>(^{100}\text{Mo} –^{130}\text{Te})</td>
<td>(T(\text{Mo}) = 4 \cdot 10^{24})</td>
<td>(6.61 \cdot 10^{24} \leq T(\text{Te}) \leq 7.20 \cdot 10^{24})</td>
<td>(2.26 \cdot 10^{24} \leq T(\text{Ge}) \leq 4.00 \cdot 10^{24})</td>
</tr>
<tr>
<td>(^{100}\text{Mo} –^{130}\text{Te})</td>
<td>(T(\text{Mo}) = 5.8 \cdot 10^{24})</td>
<td>(4.56 \cdot 10^{24} \leq T(\text{Te}) \leq 4.97 \cdot 10^{24})</td>
<td>(1.45 \cdot 10^{25} \leq T(\text{Mo}) \leq 1.78 \cdot 10^{25})</td>
</tr>
<tr>
<td>(^{100}\text{Mo} –^{130}\text{Te})</td>
<td>(T(\text{Te}) = 3 \cdot 10^{24})</td>
<td>(2.42 \cdot 10^{24} \leq T(\text{Mo}) \leq 2.63 \cdot 10^{24})</td>
<td>(2.25 \cdot 10^{25} \leq T(\text{Ge}) \leq 2.76 \cdot 10^{25})</td>
</tr>
<tr>
<td>(^{100}\text{Mo} –^{130}\text{Te})</td>
<td>(T(\text{Te}) = 1.65 \cdot 10^{25})</td>
<td>(1.33 \cdot 10^{25} \leq T(\text{Mo}) \leq 1.45 \cdot 10^{25})</td>
<td>(1.36 \cdot 10^{25} \leq T(\text{Ge}) \leq 1.82 \cdot 10^{25})</td>
</tr>
<tr>
<td>(^{100}\text{Mo} –^{130}\text{Te})</td>
<td>(T(\text{Te}) = 3 \cdot 10^{25})</td>
<td>(2.42 \cdot 10^{25} \leq T(\text{Mo}) \leq 2.63 \cdot 10^{25})</td>
<td>(7.47 \cdot 10^{25} \leq T(\text{Ge}) \leq 1.00 \cdot 10^{26})</td>
</tr>
</tbody>
</table>

“CD-Bonn, large, \(g_A = 1.0\)” NMEs: intervals change by \(\pm 5\%\);
“Argonne, large, \(g_A = 1.25 \ (1.0)\)” NMEs: intervals change by \(\pm 10\% \ (\pm 14\%)\).
$|\eta_\nu|^2$: solid lines; $|\eta_R|^2$: dashed lines.
Physical solutions - between the two vertical lines; the solutions in the grey area excluded by the lower limit $T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 1.9 \times 10^{25}$ y.
Solutions for $|\eta_\nu|^2$ (black lines) and $|\eta_R|^2$ (red lines), for given $T_1 = T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}$ yr and $T_2 = T_{1/2}^{0\nu}(^{130}\text{Te})$ and the “large basis” NMEs corresponding to: i) CD-Bonn p., $g_A = 1.25$ (solid lines), $g_A = 1$ (dashed lines) (u.l. panel); ii) CD-Bonn (solid lines) and Argonne (dashed lines) p. with $g_A = 1.25$ (u.r. panel); iii) CD-Bonn (solid lines) and Argonne (dashed lines) p. with $g_A = 1.0$ (l.l. panel); iv) Argonne p. with $g_A = 1.25$ (solid lines), $g_A = 1$ (dashed lines) (l.r. panel). The physical (positive) solutions shown with solid (dashed) lines - between the two vertical solid (dashed) lines. The horizontal dashed line - the prospective KATRIN limit $|<m>| < 0.2$ eV.
The degeneracy between the intervals of allowed values of $T_2/T_1$ (determined by the positivity conditions) corresponding to different pairs of “non-interfering” mechanisms can be lifted by using in the analysis isotopes with largely different NMEs, e.g., $^{136}Xe$ and any of $^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te$.


$T_{1/2}^{0\nu}(^{136}Xe) > 1.6 \times 10^{25} y$, 90% C.L., EXO;

$T_{1/2}^{0\nu}(^{136}Xe) > 1.9 \times 10^{25} y$, 90% C.L., KamLAND – Zen.
The relative differences between the Argonne NMEs (upper panels) and CD-Bonn NMEs (lower panels) \((M_j^{0\nu} - M_i^{0\nu})/M_i^{0\nu}\), where \(i = {^{136}Xe}\) and \(j = {^{76}Ge, {^{82}Se, {^{100}Mo, {^{130}Te}}\), for \(g_A = 1.25\) (left panel) and \(g_A = 1\) (right panel) and for three different non-interfering mechanisms: light Majorana neutrino exchange (circles), RH heavy Majorana neutrino exchange (squares) and gluino exchange (diamonds).
The conditions $|\eta_\nu|^2 > 0$ and $|\eta_R|^2 > 0$ imply for the NMEs corresponding to $g_A = 1.25 (1.0)$:

$$1.90 (1.85) \leq \frac{T_{1/2}^{0\nu}(76 Ge)}{T_{1/2}^{0\nu}(136 Xe)} \leq 2.70 (2.64) \quad \text{(Argonne NMEs)};$$

$$1.30 (1.16) \leq \frac{T_{1/2}^{0\nu}(76 Ge)}{T_{1/2}^{0\nu}(136 Xe)} \leq 2.47 (2.30) \quad \text{(CD – Bonn NMEs)}.$$

Similarly, $|\eta_\lambda|^2 > 0$ and $|\eta_R|^2 > 0$ imply:

$$2.70 (2.64) \leq \frac{T_{1/2}^{0\nu}(76 Ge)}{T_{1/2}^{0\nu}(136 Xe)} \leq 2.78 (2.67) \quad \text{(Argonne NMEs)};$$

$$1.30 (1.16) \leq \frac{T_{1/2}^{0\nu}(76 Ge)}{T_{1/2}^{0\nu}(136 Xe)} \leq 4.43 (4.25) \quad \text{(CD – Bonn NMEs)}.$$
Using the EXO limit we obtain:

\[ T^{0\nu}_{1/2}(^{76}Ge) \geq 3.03 (2.95) \times 10^{25} \text{y}, \quad \text{Argonne NMEs}; \]
\[ T^{0\nu}_{1/2}(^{76}Ge) \geq 2.08 (1.85) \times 10^{25} \text{y}, \quad \text{CD – Bonn NMEs}. \]
Two “Interfering” Mechanisms
Example: light Majorana $\nu$ and gluino exchanges

In this case for a given $(\beta\beta)_{0\nu}$ decaying $(A, Z)$,

$$
\frac{1}{T_{1/2,i} G_i^{0\nu}(E,Z)} = \frac{1}{T_{1/2,i} G_i^{0\nu}(E,Z)} = |\eta_\nu|^2 |M'_{i,\nu}|^2 + |\eta_{\lambda'}|^2 |M'_{i,\lambda'}|^2 + 2 \cos \alpha |M'_{i,\lambda'}||M'_{i,\nu}| |\eta_\nu||\eta_{\lambda'}| 
\alpha = \text{arg}(\eta_\nu \eta_{\lambda'}^{*}) \ (\text{NMEs - real}).
$$

The LNV parameters $|\eta_\nu|$, $|\eta_{\lambda'}|$ and $\cos \alpha$ - from “data” on $T_{1/2}^{0\nu}$ of three nuclei.

The solutions read:

$$
|\eta_\nu|^2 = \frac{D_1}{D}, \quad |\eta_{\lambda'}|^2 = \frac{D_2}{D}, \quad z \equiv 2 \cos \alpha |\eta_\nu||\eta_{\lambda'}| = \frac{D_3}{D},
$$
D = \begin{pmatrix}
(M_{
u}^{0\nu})^2 & (M_{\nu}^{0\nu})_{\lambda,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
(M_{\nu}^{0\nu})_{\lambda,\nu}^2 & (M_{\nu}^{0\nu})_{2,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
(M_{\nu}^{0\nu})_{3,\nu}^2 & (M_{\nu}^{0\nu})_{3,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
\end{pmatrix}, \quad D_1 = \begin{pmatrix}
1/T_1G_1 & (M_{\nu}^{0\nu})_{1,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
1/T_2G_2 & (M_{\nu}^{0\nu})_{2,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
1/T_3G_3 & (M_{\nu}^{0\nu})_{3,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
\end{pmatrix}, \quad D_2 = \begin{pmatrix}
1/T_1G_1 & (M_{\nu}^{0\nu})_{1,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
1/T_2G_2 & (M_{\nu}^{0\nu})_{2,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
1/T_3G_3 & (M_{\nu}^{0\nu})_{3,\nu}^2 & M_{\nu}^{0\nu}M_{\nu}^{0\nu}
\end{pmatrix}

Physical solutions ("positivity conditions"): 

$$|\eta\nu|^2 \geq 0, \quad |\eta_{\lambda'}|^2 \geq 0, \quad -|\eta\nu||\eta_{\lambda'}| \leq \cos \alpha|\eta\nu||\eta_{\lambda'}| \leq |\eta\nu||\eta_{\lambda'}|.$$ 

Given (i.e. having data on) $T_1$, $T_2$ + using the condition on the interference term $z = 2\cos \alpha|\eta\nu||\eta_{\lambda'}|$, determines an interval of allowed values of $T_3$.

Ranges of half-lives $T_3$ in the case of two interfering mechanisms: the light Majorana neutrino exchange and gluino exchange dominance.

<table>
<thead>
<tr>
<th>$T_{\nu}^{0\nu}$</th>
<th>$T_{\nu}^{0\nu}$</th>
<th>Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$5.99 \cdot 10^{24} \leq T(Te) \leq 7.35 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$2.46 \cdot 10^{24} \leq T(Mo) \leq 2.82 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Ge) = 10^{26}$</td>
<td>$6.30 \cdot 10^{24} \leq T(Te) \leq 6.94 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Ge) = 10^{26}$</td>
<td>$2.55 \cdot 10^{24} \leq T(Mo) \leq 2.72 \cdot 10^{24}$</td>
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<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$2.14 \cdot 10^{25} \leq T(Mo) \leq 3.31 \cdot 10^{25}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Ge) = 10^{26}$</td>
<td>$2.38 \cdot 10^{25} \leq T(Mo) \leq 2.92 \cdot 10^{25}$</td>
</tr>
</tbody>
</table>
“CD-Bonn potential, large, $g_A = 1$” NMEs

<table>
<thead>
<tr>
<th>$T_{1/2}^{0\nu} [y]$ (fixed)</th>
<th>$T_{1/2}^{0\nu} [y]$ (fixed)</th>
<th>Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Mo) = 5.8 \cdot 10^{24}$</td>
<td>$3 \cdot 10^{24} \leq T(Te) \leq 8.62 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Te) = 3 \cdot 10^{24}$</td>
<td>$2.55 \cdot 10^{24} \leq T(Mo) \leq 6.18 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Te) = 3 \cdot 10^{25}$</td>
<td>$1.33 \cdot 10^{25} \leq T(Mo) \leq 3.88 \cdot 10^{26}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Mo) = 5.8 \cdot 10^{24}$</td>
<td>$3.62 \cdot 10^{24} \leq T(Te) \leq 6.04 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Te) = 3 \cdot 10^{24}$</td>
<td>$3.11 \cdot 10^{24} \leq T(Mo) \leq 4.70 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Te) = 3 \cdot 10^{25}$</td>
<td>$2.15 \cdot 10^{25} \leq T(Mo) \leq 8.29 \cdot 10^{25}$</td>
</tr>
</tbody>
</table>

“Argonne potential, large, $g_A = 1.25$” NMEs

<table>
<thead>
<tr>
<th>$T_{1/2}^{0\nu} [y]$ (fixed)</th>
<th>$T_{1/2}^{0\nu} [y]$ (fixed)</th>
<th>Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Mo) = 5.8 \cdot 10^{24}$</td>
<td>$3 \cdot 10^{24} \leq T(Te) \leq 9.22 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Te) = 3 \cdot 10^{24}$</td>
<td>$2.55 \cdot 10^{24} \leq T(Mo) \leq 7.92 \cdot 10^{24}$</td>
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<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Te) = 3 \cdot 10^{25}$</td>
<td>$1.19 \cdot 10^{25} \leq T(Mo) \leq 2.55 \cdot 10^{27}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Mo) = 5.8 \cdot 10^{24}$</td>
<td>$3.15 \cdot 10^{24} \leq T(Te) \leq 5.85 \cdot 10^{24}$</td>
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<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Te) = 3 \cdot 10^{24}$</td>
<td>$3.25 \cdot 10^{24} \leq T(Mo) \leq 5.49 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Te) = 3 \cdot 10^{25}$</td>
<td>$2.08 \cdot 10^{25} \leq T(Mo) \leq 1.20 \cdot 10^{26}$</td>
</tr>
</tbody>
</table>

“Argonne Potential, large, $g_A = 1$” NME

<table>
<thead>
<tr>
<th>$T_{1/2}^{0\nu} [y]$ (fixed)</th>
<th>$T_{1/2}^{0\nu} [y]$ (fixed)</th>
<th>Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Mo) = 5.8 \cdot 10^{24}$</td>
<td>$3 \cdot 10^{24} \leq T(Te) \leq 1.11 \cdot 10^{25}$</td>
</tr>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Te) = 3 \cdot 10^{24}$</td>
<td>$2.63 \cdot 10^{24} \leq T(Mo) \leq 2.04 \cdot 10^{25}$</td>
</tr>
<tr>
<td>$T(Ge) = 2.23 \cdot 10^{25}$</td>
<td>$T(Te) = 3 \cdot 10^{25}$</td>
<td>$9.19 \cdot 10^{24} \leq T(Mo) \leq 2.36 \cdot 10^{26}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Mo) = 5.8 \cdot 10^{24}$</td>
<td>$3 \cdot 10^{24} \leq T(Te) \leq 5.07 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Te) = 3 \cdot 10^{24}$</td>
<td>$3.82 \cdot 10^{24} \leq T(Mo) \leq 9.44 \cdot 10^{24}$</td>
</tr>
<tr>
<td>$T(Ge) = 10^{26}$</td>
<td>$T(Te) = 3 \cdot 10^{25}$</td>
<td>$1.96 \cdot 10^{25} \leq T(Mo) \leq 6.54 \cdot 10^{26}$</td>
</tr>
</tbody>
</table>
$|\eta_\nu|^2 \times 10^{10}$: solid lines; $|\eta_\nu|^2 \times 10^{14}$: dashed lines. All solutions: $\cos \alpha \approx -1$.

Allowed regions (physical solutions) - white areas;
the solutions in the grey (blue) areas - excluded.

The horizontal solid (dashed) line - the Moscow-Mainz limit $|<m>| < 2.3$ eV
(the prospective KATRIN limit $|<m>| < 0.2$ eV).
A case of constructive interference: $\cos \alpha > 0$.

$|\eta_\nu|^2 \times 10^{10}$ (solid line), $|\eta_\lambda|^2 \times 10^{14}$ (dashed line) and $z \times 10^{12} = 2 \cos \alpha |\eta_\nu||\eta_\lambda| \times 10^{12}$ (dot-dashed line).
Conditions for $|\eta_\nu|^2 > 0$, $|\eta_\lambda'|^2 > 0$ and $z = 0$ (no int.), or $z > 0$ (constructive int.), or $z < 0$ (destructive int.).

The general conditions were derived. Below - the conditions for “CD-Bonn, large, $g_a = 1.24$” NMEs.

Given $T_1$, $|\eta_\nu|^2 > 0$, $|\eta_\lambda'|^2 > 0$, $z > 0$:

$$z > 0 : \begin{cases} 0.14 T_1 < T_2 \leq 0.16 T_1, \\ 0.16 T_1 < T_2 < 0.18 T_1, \\ \frac{4.44 T_1 T_2}{3.74 T_1 - 0.93 T_2} \leq T_3 \leq \frac{2.10 T_1 T_2}{1.78 T_1 - 0.47 T_2}; \\ \frac{4.44 T_1 T_2}{3.74 T_1 - 0.93 T_2} \leq T_3 \leq \frac{4.10 T_1 T_2}{3.44 T_1 - 0.81 T_2} \end{cases}$$

Given $T_1$, $z > 0$ only if $T_2$ lies in a relatively narrow interval and $T_3$ has a value in extremely narrow intervals; a consequence of the values of $G_i$ and of the NMEs used: for the 3 nuclei considered, $|M_i - M_j| << M_i, M_j$, $|\Lambda_i - \Lambda_j| << \Lambda_i, \Lambda_j$, $i \neq j = 1, 2, 3$, and typically $|M_i - M_j|/(0.5(M_i + M_j)) \sim 10^{-1}$, $|\Lambda_i - \Lambda_j|/(0.5(\Lambda_i + \Lambda_j)) \sim (10^{-2} - 10^{-1})$, $M_i \equiv M_{i,\nu}^{0\nu}$, $\Lambda_i \equiv M_{i,\lambda'}^{0\nu}$, $i = ^{76}Ge, ^{100}Mo, ^{130}Te$. 
Given $T_1, |\eta_\nu|^2 > 0, |\eta_\lambda'|^2 > 0, z < 0$:

\[
z < 0 : \begin{cases}
T_2 \leq 0.14 T_1, \\
0.14 T_1 < T_2 \leq 0.18 T_1, \\
0.18 T_1 < T_2 < 4.23 T_1, \\
T_2 \geq 4.23 T_1
\end{cases}
\]

$T_3 \leq \frac{2.10 T_1 T_2}{1.78 T_1 - 0.47 T_2};$

$T_3 \leq \frac{3.74 T_1 - 0.93 T_2}{4.10 T_1 T_2};$

$T_3 \leq \frac{3.44 T_1 - 0.81 T_2}{4.10 T_1 T_2};$

$T_3 > 0.$

The intervals of values of $T_2$ and $T_3$ - very different from those corresponding to the cases of two “non-interfering” mechanisms (the only exception - the second set of intervals which partially overlap with the latter).

This difference can allow to discriminate experimentally between the two possibilities of $(\beta\beta)_{0\nu}$-decay being triggered by two “non-interfering” mechanisms or by two “destructively interfering” mechanisms.

Given $T_1, |\eta_\nu|^2 = 0, |\eta_\lambda'|^2 > 0 (z = 0)$:

$T_2 = 0.14 T_1, \quad T_3 = \frac{2.10 T_1 T_2}{1.78 T_1 - 0.47 T_2} \approx 0.18 T_1.$

Given $T_1, |\eta_\nu|^2 > 0, |\eta_\lambda'|^2 = 0 (z = 0)$:

$T_2 = 0.18 T_1, \quad T_3 = \frac{4.10 T_1 T_2}{3.44 T_1 - 0.81 T_2} \approx 0.22 T_1.$
Additional consequence of “positivity” and “interference” conditions.

Given $T^{0\nu}_{1/2}$ of one isotope, say of $^{76}\text{Ge}$ ($T_1$) + an experimental lower bound on the $T^{0\nu}_{1/2}$ of a 2nd isotope, e.g., of $^{130}\text{Te}$ ($T_3$), the conditions imply a constraint on the $T^{0\nu}_{1/2}$ of any 3rd isotope, say of $^{100}\text{Mo}$ ($T_2$).

The constraint depends noticeably on the type of the two “interfering” mechanisms generating the $(\beta\beta)^{0\nu}$-decay and can be used, in principle, to discriminate between the different possible pairs of “interfering” mechanisms.
Example:  \( T_1 = 2.23 \times 10^{25} \text{ y} \ (^{76}\text{Ge}) \),  \( T_3 > 3.0 \times 10^{24} \text{ y} \ (^{130}\text{Te}) \), constraint on  \( T_2 \) (\(^{100}\text{Mo}\)); “CD-bonn (Argonne), large,  \( g_A = 1.25\)” NMEs used.

**Light Neutrino and gluino exchange mechanisms:**

\[
T_2 \equiv T_{1/2}^{0\nu}(^{100}\text{Mo}) > 2.46 \ (2.47) \times 10^{24} \text{ y}.
\]

*(Increasing the value of  \( T_{1/2}^{0\nu}(^{76}\text{Ge}) \) leads to the increasing of the value of the lower limit.)*

**Light Neutrino and LH Heavy neutrino exchanges:**

\[
T_{1/2}^{0\nu}(^{100}\text{Mo}) > 2.78 \ (2.68) \times 10^{24} \text{ y}.
\]

*(The value of the lower limit increases with the increasing of the value of  \( T_{1/2}^{0\nu}(^{76}\text{Ge}) \).)*

**Squarks-neutrino and gluino exchange mechanisms:**

\[
T_{1/2}^{0\nu}(^{100}\text{Mo}) > 7.92 \ (22.1) \times 10^{23} \text{ y}.
\]

*(For larger values of  \( T_{1/2}^{0\nu}(^{76}\text{Ge}) \), this lower bound assumes larger values.)*
LH Heavy neutrino and gluino exchange mechanisms:

\[ 1.36 \times 10^{24} \text{ y} < T_{1/2}^{0\nu}(^{100}\text{Mo}) < 3.42 \times 10^{24} \text{ y}. \]

Increasing the value of \( T_{1/2}^{0\nu}(^{76}\text{Ge}) \) leads to a shift of the interval to larger values; for a sufficiently large \( T_{1/2}^{0\nu}(^{76}\text{Ge}) > 10^{26} \text{ y} \) - only a lower bound on \( T_{1/2}^{0\nu}(^{100}\text{Mo}) \). Using the NMEs derived with the Argonne potential - only a lower bound: \( T_{1/2}^{0\nu}(^{100}\text{Mo}) > 5.97 \times 10^{23} \text{ y} \). The difference between the results obtained with the two sets of NMEs can be traced to fact that the determinant \( D \), calculated with the second set of NMEs, has opposite sign to that, calculated with the first set of NMEs. As a consequence, the dependence of the physical solutions for \( |\eta_N^L|^2 \) and \( |\eta_\lambda|^2 \) on \( T_1, T_2 \) and \( T_3 \) in the two cases of NMEs is very different.

The constraints thus obtained can be used, e.g., to exclude some of the possible cases of two “interfering” mechanisms inducing the \((\beta\beta)_{0\nu}\)-decay: if, e.g., it is confirmed that \( T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25} \text{ y} \), and in addition it is established that \( T_{1/2}^{0\nu}(^{100}\text{Mo}) \leq 10^{24} \text{ y} \), that combined with the experimental lower limit on \( T_{1/2}^{0\nu}(^{130}\text{Te}) \) would rule out i) the light neutrino and gluino exchanges, and ii) the light neutrino and LH heavy neutrino exchanges, as possible mechanisms generating the \((\beta\beta)_{0\nu}\)-decay.
Conclusions.

If the decay \((A, Z) \rightarrow (A, Z + 2) + e^- + e^- ((\beta\beta)_{0\nu}-\text{decay})\) will be observed, the questions will inevitably arise:

Which mechanism is triggering the decay?
How many mechanisms are involved?

Discussed how one possibly can answer these questions.

- The measurements of the \((\beta\beta)_{0\nu}\)-decay half-lives with rather high precision and the knowledge of the relevant NMEs with relatively small uncertainties is crucial for establishing that more than one mechanisms are operative in \((\beta\beta)_{0\nu}\)-decay.
- The method considered can be generalised to the case of more than two \((\beta\beta)_{0\nu}\)-decay mechanisms.
- It allows to treat the cases of CP conserving and CP nonconserving couplings generating the \((\beta\beta)_{0\nu}\)-decay in a unique way.
To Frank and Ettore:

Happy Anniversary!

Thank you for continuing to be the driving force in the field of $(\beta\beta)_{0\nu}$-decay research and inspiration for the younger generation of researchers working in this field.

My best wishes for personal happiness and many professional successes!